Problems are mostly (but not all) from Chapter 2 of the 3rd edition of the text. Problems coming from the textbook have their problem number listed below.

If you are not using the 3rd edition, be careful — question numbers may not agree.

1. **(2.4)** Consider the open sentence $P(x) : x(x-1) = 6$ over the domain $\mathbb{R}$.
   (a) For what values of x is $P(x)$ a true statement?
   (b) For what values of x is $P(x)$ a false statement?

2. **(2.14)** State the negation of each of the following statements.
   (a) At least two of my library books are overdue.
   (c) No one expected that to happen.
   (e) It's surprising that there were two students who received the same exam score.

3. **(2.18)** Let $S = \{1, 2, 3, 4, 5, 6\}$ and let

$$P(A) : A \cap \{2, 4, 6\} = \emptyset \quad \text{and} \quad Q(A) : A \neq \emptyset$$

   be open sentences over the domain $\mathcal{P}(S)$.
   (a) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.
   (b) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \vee (\sim Q(A))$ is true.

4. **(2.23)** Suppose that $\{S_1, S_2\}$ is a partition of the set S and $x \in S$. Which of the following are true? (Multiple are possible)
   (a) If we know that $x \notin S_1$, then x must belong to S_2.
   (b) It's possible that $x \notin S_1$ and $x \notin S_2$.
   (c) $x \notin S_1$ or $x \notin S_2$.
   (d) $x \in S_1$ or $x \in S_2$.
   (e) It's possible that $x \in S_1$ and $x \in S_2$.

5. **(2.34)** Each of the following describes an implication. Write the implication in the form 'if, then.'
   (a) Any point on the straight line with equation $2y + x - 3 = 0$ whose x-coordinate is an integer also has an integer for its y-coordinate.
   (b) The square of any odd integer is odd.
   (c) Let $n \in \mathbb{Z}$. Whenever $3n + 7$ is even, n is odd.

6. **(2.42)** Determine all values of n in the domain $S = \{2, 3, 4\}$ for which the following is a true statement: The integer $\frac{n(n-1)}{2}$ is odd if and only if the integer $\frac{n(n+1)}{2}$ is even.

7. **(2.54)** For statements P and Q, show that $((\sim Q) \implies (P \wedge (\sim P)))$ and Q are logically equivalent.

8. **(2.60)** Consider the implication: If x and y are even, then xy is even.

   (c) State the implication as a disjunction. (In particular, without the use of the word 'and' or the symbol $\wedge$.)

9. **(2.75)** Consider the quantified statement

   For every $s \in S$ and every $t \in T$, $st - 2$ is prime.

   where the domain of the variables s and t is $S = T = \{3, 5, 11\}$.

   (a) Express this quantified statement in symbols.

   (b) Is the quantified statement in (a) true or false? Explain.

   (c) Express the negation of the quantified statement in (a) in symbols.

10. **(2.86)** Given the implication $(Q \vee R) \implies (\sim P)$ is false and Q is false, determine the truth values of R and P.

11. Consider the open sentence $P(a, b) : |a - b| < 2$ where the domain of a is $A = \{2, 5, 8\}$ and the domain of b is $B = \{3, 4, 7\}$.

    (a) State the quantified statement $\forall a \in A, \exists b \in B, P(a, b)$ in words.

    (b) Prove that the quantified statement $\forall a \in A, \exists b \in B, P(a, b)$ is true.

    (c) Prove that the quantified statement $\exists b \in B, \forall a \in A, P(a, b)$ is false.