Some problems come from Chapter 4 of the 3rd edition of the text. Problems coming from the textbook have their problem number listed below.

If you are not using the 3rd edition, be careful — question numbers may not agree.

MQ? Let $n \in \mathbb{Z}$.

(a) Prove that if $n \equiv 5 \pmod{12}$, then $n \equiv 1 \pmod 4$ and $n \equiv 2 \pmod 3$.

(b) Prove that if $n \equiv 1 \pmod 4$ and $n \equiv 2 \pmod 3$, then $n \equiv 5 \pmod{12}$.

MQ? Prove that for all $m, n \in \mathbb{Z}$, $4|(3m + 5n) \iff 4|(m - n)$.

? Let $n \in \mathbb{Z}$.

(a) Prove that $3|n^2$ if and only if $3|n$.

(b) Prove that $27|n^2$ if and only if $9|n$.

? (Let $m, n \in \mathbb{Z}$. Prove that if $m^2 - 5mn + n^2 = 12$, then $m \equiv n \pmod 3$.)

1. Prove or disprove: For all $n \in \mathbb{Z}$,

$$((2 \nmid n) \wedge (3 \nmid n)) \implies \exists m \in \mathbb{Z}, mn \equiv 1 \pmod 6$$

2. Prove: For all $n \in \mathbb{Z}$,

$$n \equiv 3 \pmod 4 \implies \sim (\exists a, b \in \mathbb{Z}, a^2 + b^2 = n)$$

3. **(4.64)** For sets A and B, find a necessary and sufficient condition for $(A \times B) \cap (B \times A) = \emptyset$. Then prove that this condition is necessary and sufficient.

4. (a) Prove that

$$\{(x, y) \in \mathbb{R}^2 : 2y = x^2\} \cup \{(x, y) \in \mathbb{R}^2 : y^2 = x^3 - x\}$$
$$= \{(x, y) \in \mathbb{R}^2 : (2y - x^2)(y^2 - x^3 + x) = 0\}$$

You may assume that a product of two numbers is zero if and only if the first number is zero or the second number is zero.

(b) Prove that

$$\{(x, y) \in \mathbb{R}^2 : 2y = x^2\} \cap \{(x, y) \in \mathbb{R}^2 : y^2 = x^3 - x\} \subseteq \{(x, y) \in \mathbb{R}^2 : \frac{x^4}{4} = x^3 - x\}$$

(c) Part (b) is *not* an equality of sets - explain why.