1. Find a formula for $1+4+7+\ldots+(3n-2)$ for positive integers n, and then verify your formula by mathematical induction.

2. Prove that $3^n > n^2$ for every positive integer n.

3. Show that $5 \mid n^5 - n$ for every natural number n.

4. Prove that $(1+2+\ldots+n)^2 = 1^3+2^3+\ldots+n^3$ for every $n \in \mathbb{N}$.

5. a) In Mathematical Proof, Chapter 6.1 we saw that $1^2+2^2+\ldots+n^2$ is the number of squares in an $n \times n$ "chess board" composed of n^2 1×1 squares. What does $1^3+2^3+3^3+\ldots+n^3$ represent geometrically?

   b) Use mathematical induction to prove that $1^3+2^3+3^3+\ +n^3 = \frac{n^2(n+1)^2}{4}$ for every positive integer n.

6. Consider the open sentence $P(n): 9+13+\ldots+(4n+5) = \frac{4n^2+14n+1}{2}$, where $n \in \mathbb{N}$.

   a) Verify the implication $P(k) \Rightarrow P(k+1)$ for an arbitrary positive integer k.

   b) Is $\forall n \in \mathbb{N}$, $P(n)$ true?

7. Consider the sequence F_1, F_2, F_3, ... , where $F_1 = 1$, $F_2 = 1$, and $F_k = F_{k-2} + F_{k-1}$. The terms of this sequence are called Fibonacci numbers.

   a) What are the first 5 Fibonacci numbers?

   b) For all $n \in \mathbb{N}$, prove that $2|F_n$ if and only if $3|n$.

   c) For all $n \in \mathbb{N}$, prove that $F_1+F_2+\ldots+F_n = F_{n+2}-1$.

   d) For every $n \in \mathbb{N}$, let T_n be the number of ways to break n into a sum of a sequence of 1's and 2's. For example, here are the possibilities listed for $n = 3$.

   $$3 = 2+1 \qquad 3 = 1+2 \qquad 3 = 1+1+1$$

   Prove that $\forall n \in \mathbb{N}$, $T_n = F_{n+1}$.

8. Prove that for every positive integer n, there exists an integer x_n such that $x_n^2 \equiv 14 \pmod{5^n}$.