

1. For the given subset A_i of \mathbb{R} and the relation R_i ($1 \leq i \leq 3$) from A_i to \mathbb{R} , determine whether R_i is a function from A_i to \mathbb{R} .
 - (a) $A_1 = \mathbb{R}$, $R_1 = \{(x, y) : x \in A_1, y = 4x - 3\}$.
 - (b) $A_2 = [0, \infty)$, $R_2 = \{(x, y) : x \in A_2, (y + 2)^2 = x\}$.
 - (c) $A_3 = \mathbb{R}$, $R_1 = \{(x, y) : x \in A_3, (x + y)^2 = 4\}$.
 2. In each of the following, a function $f_i : A_i \rightarrow \mathbb{R}$ ($i = 3, 5$) is defined, where the domain A_i consists of all real numbers x for which $f_i(x)$ is defined. In each case, determine the domain A_i and the range of f_i .
 - (a) $f_3 = \sqrt{3x - 1}$
 - (b) $f_5 = \frac{x}{x-3}$.
 3. Let $A = \{5, 6\}$, $B = \{5, 7, 8\}$ and $S = \{n : n \geq 3 \text{ is an odd integer}\}$. A relation R from $A \times B$ to S is defined as $(a, b)Rs$ if $s|(a + b)$. Is R a function from $A \times B$ to S ?
 4. For a function $f : A \rightarrow B$ and subsets C and D of A and E and F of B , prove the following.
 - (a) $f(C \cup D) = f(C) \cup f(D)$
 - (b) $f^{-1}(E - F) = f^{-1}(E) - f^{-1}(F)$
 5.
 - (a) Give an example of two sets A and B such that $|B^A| = 8$.
 - (b) Give an example of an element in B^A for the sets A and B given in (a).
 6. A function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ is defined by $f(n) = 2n + 1$. Determine whether f is (a) injective, (b) surjective.
 7. Determine whether the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 + 4x + 9$ is (a) one-to-one, (b) onto.
 8. Give an example of a function $f : \mathbb{N} \rightarrow \mathbb{N}$ that is
 - (a) one-to-one and onto
 - (b) one-to-one but not onto
 - (c) onto but not one-to-one
 - (d) neither one-to-one nor onto.
 9. Let A , B and C be nonempty sets and let f , g and h be functions such that $f : A \rightarrow B$, $g : B \rightarrow C$ and $h : B \rightarrow C$. For each of the following, prove or disprove:
 - (a) $g \circ f = h \circ f$, then $g = h$.
 - (b) If f is one-to-one and $g \circ f = h \circ f$, then $g = h$.
 10. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = x^2 + ax + b$, where $a, b \in \mathbb{R}$. Show that f is not one-to-one. [Hint: It might be useful to consider the cases $a \neq 0$ and $a = 0$ separately.]
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11. Let $A = \mathbb{R} - \{0\}$ and let $f : A \rightarrow A$ be defined by $f(x) = 1 - \frac{1}{x}$ for all $x \in A$.
- (a) Show that $f \circ f \circ f = i_A$. (b) Determine f^{-1} .
12. For nonempty sets A and B and functions $f : A \rightarrow B$ and $g : B \rightarrow A$, suppose that $g \circ f = i_A$, the identity function on A .
- (a) Prove that f is one-to-one and g is onto.
- (b) Show that f need not be onto.
- (c) Show that g need not be one-to-one.
- (d) Prove that if f is onto, then g is one-to-one.
- (e) Prove that if g is one-to-one, then f is onto.
- (f) Combine the results in (d) and (e) into a single statement.